Exploring Interval-Valued Fermatean Neutrosophic Tactics for Empowering AI-Driven Financial Risk Frameworks: Compliance Automation, Fraud Detection, and Beyond

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***Abstract*—The financial risk evaluation process, which includes the investigation of the risks related to loans, funding, and trading activities in economic decisions, is greatly utilized in advanced financial systems. However, nowadays in an inconsistent, fast, and digitalized world, conventional risk models are inadequate, particularly when there is ambiguity, inconsistency, or incomplete information provided in the data. In such scenarios, the interference of Artificial Intelligence (AI) is playing a crucial role. Not only are the large data sets dealt with by AI, but also the decision-making processes can be enhanced. Conventional mathematical tools cannot analyze the discipline whose limits are complex, ambiguous, and indeterminate. In this research article, to recognize and analyze these mathematical disciplines, Interval-Valued Fermatean Neutrosophic Numbers (I-VFNNs) are used, an argumentation of modern fuzzy logic. I-VFNNs are particularly mapped out for such circumstances where the information in the data contains uncertainty, ambiguity, or inconsistency. We have used Interval-Valued Fermatean Neutrosophic Numbers (I-VFNNs), an extension of modern fuzzy logic, to identify and analyze these mathematical constraints. In this article, firstly, a list of essential aspects is composed that are affected by Artificial Intelligence in financial risk models, like fraud detection and prevention, stress testing and scenario simulation, automation of regulatory compliance, behavioral risk analysis, enhanced predictive accuracy, dynamic risk modeling, and real-time risk monitoring, etc. All these components we visualized as mathematical disciplines, which are non-probabilistic, irregular, and multifaceted in nature. With the help of I-VFNNs, we portrayed these disciplines in the guise of numbers and demonstrated their influence, intensity, and indeterminacy in accordance with the impact of Artificial intelligence. The results demonstrate that I-VFNNs not only composed ambiguity superiorly, but also refined the disparity between various risk factors. In general, not only are modern ways to constructively assess financial risk models based on Artificial Intelligence (AI) developed, but new approaches in the inspection of indeterminate data using I-VFNNs are furnished in this study. By virtue of this model, in financial organizations, better decisions can be made, accelerate the recognition of risks, and put down the right consideration even in uncertain conditions. In the future, this model can also contribute to other innovative research areas such as international financial policies, large investments, and insurance institutions.**

Keywords*—Financial Risk Evaluation; Logical Fuzzy Operators; Interval-Valued Fermatean Neutrosophic Numbers (I-VFNNs); Multi-Criteria Decision-Making; Artificial Intelligence

I. Introduction

In a world full of rising technologies, where traditional methods now look fragile, this modification is being made in the global financial system. Specifically, financial risk evaluation is a necessary pillar for financial establishments, insurance companies, and banks [1]. In particular, there are upgraded stimulations in financial risk assessment, which is a fundamental pillar for financial institutions, banks, insurance companies, and investors. A couple of the main constituents are unpredictable changes in geography and politics, fast digital transactions, increasing interrelationship between economies, and changing consumers [2]. Now there is more complexity, ambiguity, and uncertainty in financial risk, as conventional analytical models often call upon predetermined suppositions and static data; therefore, they cannot be considered adequate to constructively address these provocations. Pictorial representations of financial risk assessment are given in the Fig. 1. In such circumstances, in the field of financial risk assessment, Artificial Intelligence (AI) has made remarkable progress [3].

Models powered by Artificial Intelligence, for instance, in deep learning and machine learning, the complexity in patterns and fashion is determined by assessing large data sets, which cannot be done by conventional models. With the assistance of Artificial Intelligence, fraud detection, and real-time risk monitoring, forecast accuracy can be improved [4]. In the assessment of financial risk, numerous new defining traits have been added by the implementation of Artificial Intelligence (AI). For instance, potentiality of fraud can be determined, surveillance of risk has become feasible, the automated method is being used for administrative conformity, forecast of fraud have become more precise and dynamic, etc. as these factors have complexity, ambiguity and uncertainty, it is not possible to grasp the complete meaning through conventional models, this is the reason why these components have been contemplated as "mathematical restraint" [5]. With the assistance of an updated mathematical structure named Interval-valued Fermatean Neutrosophic

Numbers (I-VFNNs), one can diagnose, categorize, and examine the efficiency of these restraints. An updated form of corporal and neuroscience theory is I-VFNNs, which divides any constituent into three algebraic elements (membership, non-membership, and indeterminacy) [6].

In this research article, by the use of I-VFNNs, a combined effort is made to indicate financial risk factors induced by AI in an algebraic way. With the assistance of this model, one can understand how AI components lead the way to reduce financial risk, and by which components, complexity, and ambiguity in risk are increased. Furthermore, not only are the numerical applications of I-VFNNs demonstrated, but also the aggregation of artificial intelligence and fuzzy logic is demonstrated to sustain a new distinguishing system in the world of finance. Through the aid of this system, better decisions can be made, risks can be identified in advance, and uncertain conditions can be effectively responded to in financial institutions. This introduction aims to introduce the observer to the significance of this research, its motive, and the fundamental frameworks that are used in it. This research may also fascinate individuals who are concerned with implementing fuzzy logic and artificial intelligence into the monetary system and are looking forward to models in which uncertainty, inconsistency, and arising financial factors can be modeled numerically. The best substructure, in contrast to conventional techniques, is provided by Modern fuzzy mathematical models like I-VFNNs, by which not only constituents affected by Artificial intelligence are designed. Rather, they activate practical and effective decisions based on them.

Fig. 1. Financial risk assessment

Binary logic and conventional set theory use crisp numbers, that is, an element either a part of a set or not. However, in several real-world incidents, we cannot express them as purely yes-no or true-false terms, such as uncertainty in measurements, linguistic assessments, etc. To provide a mathematical framework to handle imprecision, ambiguity, and inconsistency, Zadeh [7] invented fuzzy sets in 1965. Fuzzy theory reduces the disparity that occurs in crisp numbers by allowing membership grades of each element in the set. When data is uncertain or incomplete, the capability of fuzzy theory makes it compulsory for dealing with complex systems and the decision-making process. Fuzzy set theory has wide applications across many fields, such as image processing, decision analysis, control systems, medical diagnosis, robotics, etc., where data is uncertain and imprecise [8]-[17]. Fuzzy sets deal with vague concepts in a more effective way than conventional sets by allowing the membership grade for every member of the set. Regardless of all these benefits, fuzzy sets can only handle the membership grade to elements and do not deal with the non-membership degree. This can be restrictive when the separate degrees have to be chosen for support and opposition under the available evidence.

In many real-world scenarios, experts may be partly confident about an element belonging to a set and partly confident about an element not belonging to a set, with uncertainty left behind. To bridge this gap, in 1986, Atanassov [18] created a more flexible and broad range approach named Intuitionistic fuzzy sets (IFSs). An Intuitionistic fuzzy set, which is actually an extended form of a fuzzy set, is described by membership $\vartheta(x)$ and non-membership $\varphi(x)$ functions, taking values within the interval [0,1] fulfilling the condition $0 \leq \vartheta(x) + \varphi(x) \leq 1$. Atanassov also introduced the hesitancy grade for the IFS to handle the hesitation factor arising in various real-life challenges, allowing the decision makers to better assess the situations and generate robust results. In comparison with classical fuzzy sets, IFSs provide more informative and resilient models [19], [20]. However, in decision-making challenges in real-world issues, it can be considered overly conservative to satisfy the condition that the sum of the membership function and the non-membership function will be less than 1. It bounds the range of acceptable membership and non-membership functions and may not completely handle the phenomena where both positive and negative testimonies are of a high degree. To master this limitation, Yager [21] and his colleagues proposed Pythagorean fuzzy sets (PyFSs), replacing the linear condition of IFSs with the quadratic one, that is, $0 \leq (\vartheta(x))^2 + (\varphi(x))^2 \leq 1$. This formulation extends the possible region of membership and non-membership values, which provides more resilience and effectiveness to model more complex decision-making problems [22], [23].

Although PyFSs extend the domain of permissible values as compared to IFSs, they are still bounded by the square-sum condition. In scenarios where even stronger positive and negative evidence are involved in expert assessments, it can still be considered too restrictive while modeling flexibility. To overcome this quadratic restriction of Pythagorean fuzzy sets, a more flexible framework referred to as fermatean fuzzy sets (FFSs) was created in 2019 by Senapati and Yager [24]. They further generalized the quadratic condition of PyFSs by a cubic one, that is, $0 \leq (\vartheta(x))^3 + (\varphi(x))^3 \leq 1$, providing a larger hesitancy grade. This cubic order framework deals with uncertainty that is more complex and decision-making processes with more enormous support and opposition [25], [26]. But still, FFSs only deal with uncertainty using only two variables and a hesitancy grade.

However, in real-world problems, information is not limited only to acceptance and rejection regions with a hesitancy grade; we need to express truth grade, falsity grade, and neutral (indeterminacy) grade. To deal with this, Smarandache [27] expanded the fuzzy models by including the three components $\langle \mathcal{T}(x), \mathcal{I}(x), \mathcal{F}(x) \rangle$ and introduced the Neutrosophic set (NS). Where $\langle \mathcal{T}(x), \mathcal{I}(x), \mathcal{F}(x) \rangle$ describe the truth grade, neutral grade, and falsity grade, where each of these can take values in the interval [0,1]. A neutrosophic set provides more flexibility to handle information that is imprecise, inconsistent, and uncertain [28], [29]. Integrating the comprehensive uncertainty modeling of neutrosophic sets with a broader admissible domain of fermatean fuzzy sets, neutrosophic fermatean sets (NFSs) are introduced [30]. This framewor combines the three functions of truth, neutrality, and falsity, along with the cubic fermatean condition that is, $0 \leq \mathcal{T}(x)^3 + \mathcal{I}(x)^3 + \mathcal{F}(x)^3 \leq 3$.

To have a more refined tool to portray the uncertain data, we have merged the traits of neutrosophic logic and fermatean fuzzy sets, allowing researchers to deal with inconsistency and varying information in complex circumstances [31], [32]. A more extravagant way can be offered by interval-valued fermatean neutrosophic sets, by extending the idea of neutrosophic theory and interval-valued fermatean fuzzy sets to deal with uncertainty in a better way [6]. The representation of I-VFNSs in intervals makes the model appropriate for decision-making and other scenarios that contain data of high variance, and incorporates both the researchers' assessments and hesitancy due to the varying information. I-VFNSs provide researches with a more widely applicable, flexible, and reliable approach to handle the uncertainty present in real-life problems [33]-[37].

In this research article, the aim is to offer a novel approach based on the incorporation of I-VFNS with logical operators. A structured algorithm for MCDM is presented to better find the solutions to different intricate challenges. The numerical implementation of the proposed MCDM approach is also given to validate its applicability. Also, a practical case study on a significant and sensitive topic, namely financial risk evaluation, is provided to show the implementation of the presented technique in real-life problems.

A. Structure of the Research Paper

The structure of the research article is elaborated in Section II where important concepts with their respective operations are given. Section III gives a theoretical overview of the impact of AI on financial risk dynamics. We developed a novel MCDM approach based on logical operators in Section IV. Numerical presentations of the proposed approach are elaborated in Section V. A comparative study of the proposed method with the previous approaches is conducted in Section VI. A brief conclusion with future directions and limitations of our research study presented in Section VII. Also, a flow diagram of the research article is illustrated in Fig. 2.

B. Key Findings

Our study provides the following noteworthy outcomes regarding the advantages of implementing interval-valued fermatean neutrosophic (I-VFN) methods to compliance fraud detection, risk assessment, and automation.

- By implementing interval-valued fermatean neutrosophic (I-VFN) sets, simultaneous truth, indeterminacy, and falsity in financial data, such as dealing with incomplete market data, conflicting fraud indicators can be captured better than other fuzzy frameworks.
- By decomposing outcomes into truth, indeterminacy, and falsity degrees, it provided a definable justification for automated decisions.
- The I-VFN framework offered richer scenario analysis, exposing hidden risks and reliance between financial mechanisms.
- Similar strategies have been applied successfully in anti-money laundering, specifying a wide financial relevance.

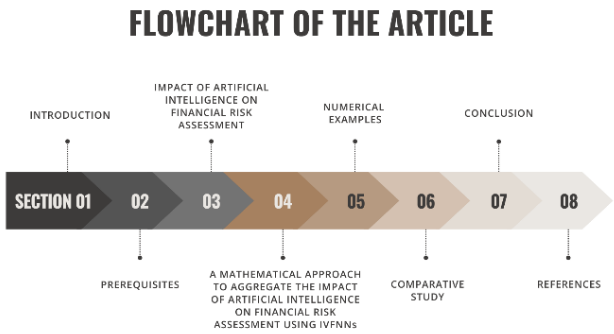

Fig. 2. Structural flow of the paper

II. PREREQUISITES

- **Definition 1: Fuzzy Set [7]**

Take a universal set $\mathcal{X}$, then a fuzzy set F is defined by a mathematical pair of values as follows,

$$F = \{(x, \vartheta(x)) : x \in \mathcal{X}\}, \tag{1}$$

where the degree of the membership function is defined by $\vartheta : \mathcal{X} \rightarrow [0,1]$, of the fuzzy set F, and the membership value of $x \in \mathcal{X}$, is shown as $\vartheta(x)$. The membership values are real numbers in the interval [0,1]. So, a fuzzy number F_N is shown as $F_N = (x, \vartheta(x))$.

- **Definition 2: Interval-Valued Fuzzy Set [38]**

Take a universal set $\mathcal{X}$, then an interval-valued fuzzy set I is defined by a mapping I: $\mathcal{X} \rightarrow$ O[0,1], where O[0,1] shows the collection of closed intervals present in [0,1] and,

$$I(x) = [\underline{I}(x), \bar{I}(x)], \tag{2}$$

Hence, we can say that I is described by two mappings that are $\underline{I}(x), \bar{I}(x): \mathcal{X} \rightarrow O[0,1]$, such that $\underline{I}(x) \leq \bar{I}(x)\ \forall\ x \in \mathcal{X}$. These mappings $\underline{I}(x), \bar{I}(x)$ are known as lower bound and upper bound of the membership function, respectively.

- **Definition 3: Intuitionistic Fuzzy Set [18]**

Take a universal set $\mathcal{X}$, an intuitionistic fuzzy set F_I is defined by the following mathematical formulation,

$$F_I = \{\langle x, \vartheta(x), \varphi(x)\rangle : x \in \mathcal{X}\}, \tag{3}$$

where the mapping is known as the membership and non-membership functions of F_I are defined by $\vartheta, \varphi: \mathcal{X} \rightarrow [0,1]$. In addition, the membership and non-membership values of $x \in \mathcal{X}$, in F_I is shown as $\vartheta(x)$ and $\varphi(x)$, respectively. These membership and non-membership values are real numbers in the interval [0,1]. Therefore, an intuitionistic fuzzy number F_{IN} is presented as $F_{IN} = (\vartheta(x), \varphi(x))$. But any pair of real numbers lying between the interval [0,1] cannot be said as an intuitionistic fuzzy number until they fulfill the following condition for existence,

$$0 \leq \vartheta(x) + \varphi(x) \leq 1.$$

- **Definition 4: Pythagorean Fuzzy Set [21]**

Take a universal set $\mathcal{X}$, a Pythagorean fuzzy set F_P has the mathematical formulation as follows,

$$F_P = \{\langle x, \vartheta(x), \varphi(x) \rangle : x \in \mathcal{X}\} \quad (4)$$

where the mappings known as the membership and non-membership functions of F_P are defined by $\vartheta, \varphi : \mathcal{X} \rightarrow [0,1]$. In addition, the membership and non-membership values of $x \in \mathcal{X}$, in the pythagorean fuzzy set F_P is shown as $\vartheta(x)$ and $\varphi(x)$, respectively. These membership and non-membership values are real numbers in the interval [0,1]. Therefore, a Pythagorean fuzzy number F_{PN} can be shown as $F_{PN} = (\vartheta(x), \varphi(x))$. But any pair of real numbers lying between the interval [0,1] cannot be said as Pythagorean fuzzy number until they fulfill the following condition for existence,

$$0 \leq (\vartheta(x))^2 + (\varphi(x))^2 \leq 1.$$

- **Definition 5: Fermatean Fuzzy Set [24]**

Take a universal set $\mathcal{X}$, a fermatean fuzzy set F_F has the mathematical formulation as follows,

$$F_F = \{\langle x, \vartheta(x), \varphi(x) \rangle : x \in \mathcal{X}\}, \quad (5)$$

where the mappings known as the membership and non-membership functions of F_F are defined by $\vartheta, \varphi : \mathcal{X} \rightarrow [0,1]$. In addition, the membership and non-membership values of $x \in \mathcal{X}$, in the fermatean fuzzy set F_F is shown as $\vartheta(x)$ and $\varphi(x)$, respectively. These membership and non-membership values are real numbers in the interval [0,1]. Therefore, a fermatean fuzzy number F_{FN} can be shown as $F_{FN} = (\vartheta(x), \varphi(x))$. But any pair of real numbers lying between the interval [0,1] cannot be said as fermatean fuzzy number until they fulfill the following condition for existence,

$$0 \leq (\vartheta(x))^3 + (\varphi(x))^3 \leq 1.$$

- **Definition 6: Neutrosophic Set [27]**

Take a universal set $\mathcal{X}$, a neutrosophic set $\mathcal{N}$ has the mathematical formulation as follows,

$$\mathcal{N} = \{\langle x, \mathcal{T}(x), \mathcal{I}(x), \mathcal{F}(x) \rangle : x \in \mathcal{X}\}, \quad (6)$$

In addition, the membership value and non-membership value of truth, neutrality and falsity of $x \in \mathcal{X}$, in neutrosophic set $\mathcal{N}$ are shown as $\mathcal{T}(x)$, $\mathcal{I}(x)$, $\mathcal{F}(x)$, respectively. These membership value and non-membership value are real numbers in the interval [0,1]. Hence, a neutrosophic number $\mathcal{N}_N$, is represented by $\mathcal{N}_N = (\mathcal{T}(x), \mathcal{I}(x), \mathcal{F}(x))$. But any pair of real numbers lying between the interval [0,1], cannot be said as neutrosophic number until they fulfill the following condition for existence,

$$0 \leq \mathcal{T}(x) + \mathcal{I}(x) + \mathcal{F}(x) \leq 3.$$

- **Definition 7: Fermatean Neutrosophic Set [30]**

Take a universal set $\mathcal{X}$, then a fermatean neutrosophic (FN) set F_N is defined by the following mathematical formulation,

$$F_N = \{\langle x, \mathcal{T}(x), \mathcal{I}(x), \mathcal{F}(x) \rangle : x \in \mathcal{X}\}, \quad (7)$$

In addition, the membership value and non-membership value of truth, neutrality and falsity of $x \in \mathcal{X}$, in the fermatean neutrosophic set F_N are shown as $\mathcal{T}(x), \mathcal{I}(x), \mathcal{F}(x)$, respectively. These membership and non-membership values are real numbers in the interval [0,1]. Therefore, a fermatean neutrosophic number F_{NN}, can be shown as $F_{NN} = (\mathcal{T}(x), \mathcal{I}(x), \mathcal{F}(x))$. But any pair of real numbers lying between the interval [0,1] cannot be said as fermatean neutrosophic number until they fulfill the following condition for existence,

$$0 \leq \mathcal{T}(x)^3 + \mathcal{I}(x)^3 + \mathcal{F}(x)^3 \leq 2.$$

- **Definition 8: Interval-Valued Fermatean Neutrosophic Set [6]**

Take a universal set $\mathcal{X}$, then an interval-valued fermatean neutrosophic (I-VFN) set F_N is defined by the following mathematical formulation,

$$I_{FN} = \left\{ \left\langle \begin{matrix} x, (\mathcal{T}_M, \mathcal{T}_N)(x), (\mathcal{I}_M, \mathcal{I}_N)(x), \\ (\mathcal{F}_M, \mathcal{F}_N)(x) \end{matrix} \right\rangle : x \in \mathcal{X} \right\}, \quad (8)$$

In addition, the membership value and non-membership value of truth, neutrality and falsity of $x \in \mathcal{X}$, in the fermatean neutrosophic set I_{FN} are shown as ordered pairs $(\mathcal{T}_M, \mathcal{T}_N)(x) \in [0,1]$, $(\mathcal{I}_M, \mathcal{I}_N)(x) \in [0,1]$, and $(\mathcal{F}_M, \mathcal{F}_N)(x) \in [0,1]$, respectively. The first element of each pair is the membership degree, and the second element is the non-membership degree. The conditions for a number to be I-VFNN are defined as;

$$0 \leq \mathcal{T}_M(x)^3 + \mathcal{F}_M(x)^3 \leq 1, \text{ and } 0 \leq \mathcal{I}_M(x)^3 \leq 1.$$

also,

$$0 \leq \mathcal{T}_N(x)^3 + \mathcal{F}_N(x)^3 \leq 1, \text{ and } 0 \leq \mathcal{I}_N(x)^3 \leq 1.$$

alternatively, these conditions can be defined as;

$$0 \leq \mathcal{T}_M(x)^3 + \mathcal{I}_M(x)^3 + \mathcal{F}_M(x)^3 \leq 2.$$

and,

$$0 \leq \mathcal{T}_N(x)^3 + \mathcal{I}_N(x)^3 + \mathcal{F}_N(x)^3 \leq 2.$$

- **Definition 9: Basic operations on Interval-Valued Fermatean Neutrosophic Set [6]**

$$I_{FN_1} = \{(\mathcal{T}_{M_1}, \mathcal{T}_{N_1})(x), (\mathcal{I}_{M_1}, \mathcal{I}_{N_1})(x), (\mathcal{F}_{M_1}, \mathcal{F}_{N_1})(x)\},$$

and

$$I_{FN_2} = \{(\mathcal{T}_{M_2}, \mathcal{T}_{N_2})(x), (\mathcal{I}_{M_2}, \mathcal{I}_{N_2})(x), (\mathcal{F}_{M_2}, \mathcal{F}_{N_2})(x)\},$$

be two I-VFN numbers and $\zeta > 0$.

The operation rules are explained below,

$$I_{FN_1} \oplus I_{FN_2} = \begin{bmatrix} \begin{pmatrix} \sqrt[3]{\{\mathcal{T}_{M_1}\}^3 + \{\mathcal{T}_{M_2}\}^3 - \{\mathcal{T}_{M_1}\}^3\{\mathcal{T}_{M_2}\}^3}, \\ \sqrt[3]{\{\mathcal{T}_{N_1}\}^3 + \{\mathcal{T}_{N_2}\}^3 - \{\mathcal{T}_{N_1}\}^3\{\mathcal{T}_{N_2}\}^3} \end{pmatrix}, \\ (\mathcal{J}_{M_1}\mathcal{J}_{M_2}, \mathcal{J}_{N_1}\mathcal{J}_{N_2}), \left((\mathcal{F}_{M_1}\mathcal{F}_{M_2}, \mathcal{F}_{N_1}\mathcal{F}_{N_2})\right) \end{bmatrix}$$

$$I_{FN_1} \bigotimes I_{FN_2} = \begin{bmatrix} (\mathcal{T}_{M_1}, \mathcal{T}_{M_2}, \mathcal{T}_{N_1}, \mathcal{T}_{N_2}), \\ \begin{pmatrix} \sqrt[3]{\{\mathcal{J}_{M_1}\}^3 + \{\mathcal{J}_{M_2}\}^3 - \{\mathcal{J}_{M_1}\}^3\{\mathcal{J}_{M_2}\}^3}, \\ \sqrt[3]{\{\mathcal{J}_{N_1}\}^3 + \{\mathcal{J}_{N_2}\}^3 - \{\mathcal{J}_{N_1}\}^3\{\mathcal{J}_{N_2}\}^3} \end{pmatrix}, \\ \begin{pmatrix} \sqrt[3]{\{\mathcal{F}_{M_1}\}^3 + \{\mathcal{F}_{M_2}\}^3 - \{\mathcal{F}_{M_1}\}^3\{\mathcal{F}_{M_2}\}^3}, \\ \sqrt[3]{\{\mathcal{F}_{N_1}\}^3 + \{\mathcal{F}_{N_2}\}^3 - \{\mathcal{F}_{N_1}\}^3\{\mathcal{F}_{N_2}\}^3} \end{pmatrix} \end{bmatrix}$$

$$\zeta I_{FN_1} = \begin{bmatrix} \left(\sqrt[3]{1 - \left[1 - \{\mathcal{T}_{M_1}\}^3\right]^\zeta}, \sqrt[3]{1 - \left[1 - \{\mathcal{T}_{N_1}\}^3\right]^\zeta}\right), \\ \left(\{\mathcal{J}_{M_1}\}^3, \{\mathcal{J}_{N_1}\}^3\right), \left(\{\mathcal{F}_{M_1}\}^3, \{\mathcal{F}_{N_1}\}^3\right) \end{bmatrix}$$

$$[I_{FN_1}]^\zeta = \begin{bmatrix} \left(\{\mathcal{T}_{M_1}\}^3, \{\mathcal{T}_{N_1}\}^3\right), \\ \left(\sqrt[3]{1 - \left[1 - \{\mathcal{J}_{M_1}\}^3\right]^\zeta}, \sqrt[3]{1 - \left[1 - \{\mathcal{J}_{N_1}\}^3\right]^\zeta}\right), \\ \left(\sqrt[3]{1 - \left[1 - \{\mathcal{F}_{M_1}\}^3\right]^\zeta}, \sqrt[3]{1 - \left[1 - \{\mathcal{F}_{N_1}\}^3\right]^\zeta}\right) \end{bmatrix}$$

- **Definition 10: Score function of Interval-Valued Fermatean Neutrosophic Set [6]**
  The score function for a I-VFN number

$$I_{FN} = \{(\mathcal{T}_M, \mathcal{T}_N)(x), (\mathcal{J}_M, \mathcal{J}_N)(x), (\mathcal{F}_M, \mathcal{F}_N)(x)\},$$

was determined by *Broumi et al.* which is defined below,

$$\mathcal{S}(x) = \frac{\mathcal{T}_M{}^3 + \mathcal{T}_N{}^3 + \mathcal{J}_M{}^3 + \mathcal{J}_N{}^3 + \mathcal{F}_M{}^3 + \mathcal{F}_N{}^3}{2} \quad (9)$$

III. IMPACT OF ARTIFICIAL INTELLIGENCE ON FINANCIAL RISK ASSESSMENT

The process of financial risk assessment has been significantly altered by the evolution of artificial intelligence (AI), which facilitates swifter, more precise, and dynamic analysis of intricate data sets. AI has improved or replaced the conventional risk models, which were often static and rule-based, by continuously enhancing these systems using factors like real-time financial, market, and behavioral data. AI has simplified the process of identifying potential threats early in the time frame, such as market volatility, loan defaults, and account fraud. The main tools that AI employs are market analysis, comprehension of human language, and forecasting. In addition, by automating rules, AI reduces errors and makes checks for money laundering and validating customers more effective and efficient. Changes initiated by AI are not only changing the entire process to make better decisions, but are also allowing banks to be the first to respond to new risks arising in a digital and interconnected world. AI systems are changing the game for the assessment of financial risk across the entire industry. Let's review a few of the areas in which AI is positively impacting financial risk. Additionally, some AI-driven factors influencing financial risk evaluation are given in Fig. 3.

A. Behavioral Risk Analysis

Artificial intelligence can observe and evaluate human behavior, whether it is for an individual or for a group, when conducting a financial assessment. AI considers things such as spending patterns, online behavior, and sometimes biometric data, for instance, fingerprints, simply to conduct a risk assessment. For example, if someone suddenly makes a large number of purchases or logs in at odd hours, it indicates potential fraud. AI can also assess aspects like insider trading, and so on, by simply observing and assessing, and relying on the observed behavior. Also, the bank's fraud team is supported by AI because there is always an AI machine (or robots) that constantly observes the customers' stream of transactions and accounts with the latest technologies. In this manner, the company has a chance to identify risks that would have otherwise gone unnoticed. Using AI for risk management of financial products is very "human-natured" and enhances the speed of decision-making, also rendering a more customized overall risk management process, and reduces the burden.

B. Real-Time Risk Management

AI is a technology that allows companies to monitor their financial health in a more contemporary and efficient fashion than was previously done with account monitoring. It analyzes customer phone data not only to keep the business aware of risks associated with the customer's accounts but to notify the business as soon as possible by observing something like changes in customer behaviors, deal-making, and changes to business processes. Machines can discover odd behaviors in trades or unexpected payments, which may indicate that someone is doing something inappropriate, or even illegal. The up-to-date use of AI allows the business to alert or communicate that action as soon as possible to help limit financial loss and damage to the brand. Financial activities have historically been very complex and related; thus, the use of AI and real-time checks has become a significant asset for decision making and risk identification in an ever-changing society.

C. Dynamic Risk Modeling

The risk model of AI represents the best way to assess risks in the financial space. It is quicker and more accurate than traditional static models. Rather than rely on historical forecasts, it adapts and evolves through ongoing observation of monetary markets, consumer behavior, and the economy. This equates to regulatory compliance, reduced losses, and better decision-making in corporate environments.

D. Fraud Detection and Prevention

AI tools have the ability to analyze thousands and thousands of different transactions to identify anomalies and patterns, which users tend to follow, helping detect suspicious activity with ease. AI-empowered algorithms adapt from historical fraud data, consistently enhancing their ability to detect new advanced fraudulent schemes. Identity verification can be improved by integrating biometric recognition with behavioral analysis by AI tools, reducing the false positives. AI-powered tools allow the risk managers to intervene before the transaction is completed by blocking or alerting about the suspicious activity. Scalable fraud detection is enabled without any significant increase in costs, making it an optimal tool for digital platforms working in large-volume environments and banks.

E. Risk Compliance Automation

Compliance can be modified by Artificial Intelligence by automating repeated manual tasks like identity verification, reporting, and document validation. By using AI tools, policy upgrades can be interpreted, a large volume of administrative texts can be examined, and designated for procedure very easily. Using artificial intelligence to monitor transaction data can help detect rule-breaking, leading to a more straightforward process with little chance of error. Moreover, the AI systems would prepare audit reports on demand, with nothing being left unaccounted for, and everything being easily traceable. AI-supported automation would also enhance people's ability to monitor and anticipate risk, thus creating a more compliant environment for financial institutions, especially banks.

F. Improved Credit Scoring

AI is greatly enhancing credit scores. It is moving beyond traditional credit histories and financial components to paint a broader and clearer picture of the consumer. Traditional credit scores cannot evaluate people with thin credit files. AI, on the other hand, can examine other areas like employment history, mobile phone bills, monthly rent payments, and even social media activity, to name a few. This provides a broader and deeper understanding of your financial management skills.

G. Enhanced Predictive Accuracy

Conventional approaches rely on hypothecation and processes that are very inefficient in volatile environments. AI has the ability to analyze both structured data (e.g., financial statements, credit ratings) and unstructured data (e.g., news articles, consumer behaviors) in order to identify trends and alert you to potential problems. AI algorithms can also update in the moment, which allows for a more precise prediction of risks, such as defaults on loans, declines in the stock market, and shortages of liquidity. In other words, the system is always improving.

H. Stress Testing and Scenario Simulation

Artificial intelligence is changing the way we simulate scenarios and stress test, especially when it comes to evaluating financial risk. It offers a holistic, efficient service to banks and other financial institutions by calculating how hard they are going to be hit, if at all, when something like the possible worst-case scenario ensues (but not entirely unthinkable!). Where traditionally only single problems are dealt with at one time, AI can take many problems and permutations and muddle through them to see what comes out on the other side, also providing a much more holistic, realistic feel of the stress. AI can also respond to new risks, current scenarios, and ensure compliance with what the available regulations say the bank should be doing to consider issues like the impact of a recession, the impact of global issues, or even an epidemic on the bank's liquidity and its buffer. By meeting controlling expectations, making logical decisions in a more efficient way, and optimizing eventual plans, institutions are enabled to guarantee the financial system remains durable

Under adverse circumstances a mathematical approach to aggregate the impact of artificial intelligence on financial risk assessment using I-VFN Numbers.

In this research article, our primary objective is to suggest an advanced framework to assess the influence of Artificial Intelligence on financial risk assessment by using Interval-Valued Fermatean Neutrosophic Numbers (I-VFNNs). Here, we are going to formulate the technique by representing the key factors of AI impacting significantly on financial risk evaluation as I-VFNNs, which will enable us to minimize the imprecision and inconsistency present in the data. Therefore, we have elaborated various key factors in the Table 1 and Fig. 4, which will be utilized as I-VFN constraints for our research study, mathematical formulation, and numerical interpretation.

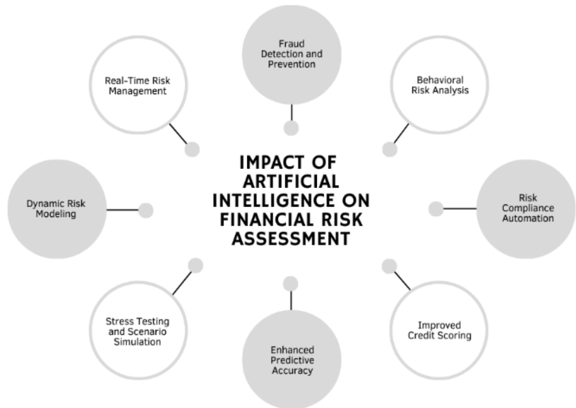

Fig. 3. Impact of AI on financial risk assessment

Table 1. Variables used in the mathematical model with dimensions

Constraint	Dimension
Real-Time Risk Monitoring ($\mathfrak{R}$)	Flag Abnormalities Continuous Scan Trigger Alerts or Automated Actions
Fraud Detection and Prevention ($\mathfrak{F}$)	Predictive Analytics Multi-Factor Validation Behavioral Profiling
Risk Compliance Automation ($\mathfrak{C}$)	Anti-Money Laundering Regulatory Change Optimization Know Your Customer (KYC)

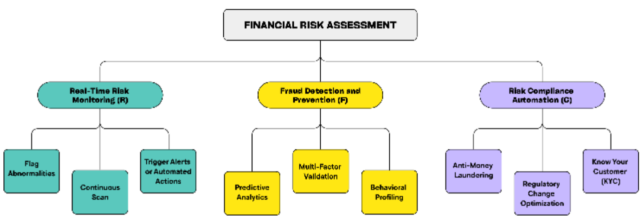

Fig. 4. Variables with their respective dimensions

As, we have previously discussed each of the key role influencing the financial risk evaluation is shown as I-VFN constraints, so we can represent the constraint for real-time risk monitoring as;

$$\mathfrak{R} = \{(\mathcal{T}_{\mathfrak{R}}^{M}, \mathcal{T}_{\mathfrak{R}}^{N}), (\mathcal{I}_{\mathfrak{R}}^{M}, \mathcal{I}_{\mathfrak{R}}^{N}), (\mathcal{F}_{\mathfrak{R}}^{M}, \mathcal{F}_{\mathfrak{R}}^{N})\}$$

for fraud detection and prevention as;

$$\mathfrak{F} = \left\{\left(\mathcal{T}_{\mathfrak{F}}^{M}, \mathcal{T}_{\mathfrak{F}}^{N}\right), \left(\mathcal{I}_{\mathfrak{F}}^{M}, \mathcal{I}_{\mathfrak{F}}^{N}\right), \left(\mathcal{F}_{\mathfrak{F}}^{M}, \mathcal{F}_{\mathfrak{F}}^{N}\right)\right\}$$

and for risk compliance automation as;

$$\mathfrak{C} = \{(\mathcal{T}_{\mathfrak{C}}^{M}, \mathcal{T}_{\mathfrak{C}}^{N}), (\mathcal{I}_{\mathfrak{C}}^{M}, \mathcal{I}_{\mathfrak{C}}^{N}), (\mathcal{F}_{\mathfrak{C}}^{M}, \mathcal{F}_{\mathfrak{C}}^{N})\}$$

Suppose that we are taking into consideration 'n' number of cases to examine the impact of AI on financial threat evaluation, so we can denote these cases as "$\mathcal{A}_i$" where $i = 1,2,3,\ldots,n$. Thereby, the notation for all these variables we have taken will be changed respectively as, for real-time risk monitoring, as;

$$\mathfrak{R}_i = \left\{\left(\mathcal{T}_{\mathfrak{R}_i}^{M}, \mathcal{T}_{\mathfrak{R}_i}^{N}\right), \left(\mathcal{I}_{\mathfrak{R}_i}^{M}, \mathcal{I}_{\mathfrak{R}_i}^{N}\right), \left(\mathcal{F}_{\mathfrak{R}_i}^{M}, \mathcal{F}_{\mathfrak{R}_i}^{N}\right)\right\}$$

for fraud detection and prevention as;

$$\mathfrak{F}_i = \left\{\left(\mathcal{T}_{\mathfrak{F}_i}^{M}, \mathcal{T}_{\mathfrak{F}_i}^{N}\right), \left(\mathcal{I}_{\mathfrak{F}_i}^{M}, \mathcal{I}_{\mathfrak{F}_i}^{N}\right), \left(\mathcal{F}_{\mathfrak{F}_i}^{M}, \mathcal{F}_{\mathfrak{F}_i}^{N}\right)\right\}$$

and for risk compliance automation as;

$$\mathfrak{C}_i = \left\{\left(\mathcal{T}_{\mathfrak{C}_i}^{M}, \mathcal{T}_{\mathfrak{C}_i}^{N}\right), \left(\mathcal{I}_{\mathfrak{C}_i}^{M}, \mathcal{I}_{\mathfrak{C}_i}^{N}\right), \left(\mathcal{F}_{\mathfrak{C}_i}^{M}, \mathcal{F}_{\mathfrak{C}_i}^{N}\right)\right\}$$

Now to examine this evaluation, we have gathered a team of expert financial risk managers which will give us the required information about the impact of AI on financial threat assessment by aggregating the variables $\mathfrak{R}$, $\mathfrak{F}$ and $\mathfrak{C}$. We denote these experts by 'j' with $j = 1,2,3,\ldots,m$, who provide the information in the allotted time and for each expert the variable representation is changed as, for real-time risk monitoring as;

$$\mathfrak{R}_{ij} = \left\{\left(\mathcal{T}_{\mathfrak{R}_{ij}}^{M}, \mathcal{T}_{\mathfrak{R}_{ij}}^{N}\right), \left(\mathcal{I}_{\mathfrak{R}_{ij}}^{M}, \mathcal{I}_{\mathfrak{R}_{ij}}^{N}\right), \left(\mathcal{F}_{\mathfrak{R}_{ij}}^{M}, \mathcal{F}_{\mathfrak{R}_{ij}}^{N}\right)\right\}$$

for fraud detection and prevention as;

$$\mathfrak{F}_i = \left\{\left(\mathcal{T}_{\mathfrak{F}_{ij}}^{M}, \mathcal{T}_{\mathfrak{F}_{ij}}^{N}\right), \left(\mathcal{I}_{\mathfrak{F}_{ij}}^{M}, \mathcal{I}_{\mathfrak{F}_{ij}}^{N}\right), \left(\mathcal{F}_{\mathfrak{F}_{ij}}^{M}, \mathcal{F}_{\mathfrak{F}_{ij}}^{N}\right)\right\}$$

and for risk compliance automation as;

$$\mathfrak{C}_i = \left\{\left(\mathcal{T}_{\mathfrak{C}_{ij}}^{M}, \mathcal{T}_{\mathfrak{C}_{ij}}^{N}\right), \left(\mathcal{I}_{\mathfrak{C}_{ij}}^{M}, \mathcal{I}_{\mathfrak{C}_{ij}}^{N}\right), \left(\mathcal{F}_{\mathfrak{C}_{ij}}^{M}, \mathcal{F}_{\mathfrak{C}_{ij}}^{N}\right)\right\}$$

The rank of each expert financial risk manager is measured through their level of expertise in the corresponding field and shown as weight vectors $\mathfrak{w}_j$ where $j = 1,2,3,\ldots,m$, as shown in Table 2.

Table 2. Expertise Level of Experts as Weights

Expert's Rank	Weights $(\mathfrak{w}_j)$
Very low	0.2
Low	0.36
Moderate	0.55
High	0.73
Very High	0.9

The grading of expertise level has been made purely on the basis of experience, speed, knowledge, and proficiency of the expert financial risk managers assessing the given variables in a limited amount of time. To get the results as precise, accurate, and effective as possible, we are going to approach the experts with a higher level of comprehension in the relevant fields or simply by having larger values of weight vectors. Therefore, we can accumulate the following weighted arithmetic means formulas for the variables $\mathfrak{R}$, $\mathfrak{F}$, and $\mathfrak{C}$ collected from the experts,

Gathering these weighted arithmetic means of each variable in the table will enable us to perform further evaluations on the data collected from the expert financial risk managers. Additionally, we are going to define two logical predicates based on I-VFN logic operators to further simplify the results given in (13) and (14).

$$\mathfrak{R}_i = \left\{\begin{array}{l} \left(\frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{T}_{\mathfrak{R}_{ij}}^{M}}{\sum_{j=1}^{m}\mathfrak{w}_j}, \frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{T}_{\mathfrak{R}_{ij}}^{N}}{\sum_{j=1}^{m}\mathfrak{w}_j}\right), \\ \left(\frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{I}_{\mathfrak{R}_{ij}}^{M}}{\sum_{j=1}^{m}\mathfrak{w}_j}, \frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{I}_{\mathfrak{R}_{ij}}^{N}}{\sum_{j=1}^{m}\mathfrak{w}_j}\right), \\ \left(\frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{F}_{\mathfrak{R}_{ij}}^{M}}{\sum_{j=1}^{m}\mathfrak{w}_j}, \frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{F}_{\mathfrak{R}_{ij}}^{N}}{\sum_{j=1}^{m}\mathfrak{w}_j}\right) \end{array}\right\} \quad (10)$$

$$\mathfrak{F}_i = \left\{\begin{array}{l} \left(\frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{T}_{\mathfrak{F}_{ij}}^{M}}{\sum_{j=1}^{m}\mathfrak{w}_j}, \frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{T}_{\mathfrak{F}_{ij}}^{N}}{\sum_{j=1}^{m}\mathfrak{w}_j}\right), \\ \left(\frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{I}_{\mathfrak{F}_{ij}}^{M}}{\sum_{j=1}^{m}\mathfrak{w}_j}, \frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{I}_{\mathfrak{F}_{ij}}^{N}}{\sum_{j=1}^{m}\mathfrak{w}_j}\right), \\ \left(\frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{F}_{\mathfrak{F}_{ij}}^{M}}{\sum_{j=1}^{m}\mathfrak{w}_j}, \frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{F}_{\mathfrak{F}_{ij}}^{N}}{\sum_{j=1}^{m}\mathfrak{w}_j}\right) \end{array}\right\} \quad (11)$$

$$\mathfrak{C}_i = \left\{\begin{array}{l} \left(\frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{T}_{\mathfrak{C}_{ij}}^{M}}{\sum_{j=1}^{m}\mathfrak{w}_j}, \frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{T}_{\mathfrak{C}_{ij}}^{N}}{\sum_{j=1}^{m}\mathfrak{w}_j}\right), \\ \left(\frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{I}_{\mathfrak{C}_{ij}}^{M}}{\sum_{j=1}^{m}\mathfrak{w}_j}, \frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{I}_{\mathfrak{C}_{ij}}^{N}}{\sum_{j=1}^{m}\mathfrak{w}_j}\right), \\ \left(\frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{F}_{\mathfrak{C}_{ij}}^{M}}{\sum_{j=1}^{m}\mathfrak{w}_j}, \frac{\sum_{j=1}^{m}\mathfrak{w}_j\mathcal{F}_{\mathfrak{C}_{ij}}^{N}}{\sum_{j=1}^{m}\mathfrak{w}_j}\right) \end{array}\right\} \quad (12)$$

$$\mathcal{E} = \bigsqcup_{\forall\, i\, \in \mathcal{A}}\{(\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i) \wedge_{\mathcal{A}} (\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i \rightarrow_{\mathcal{A}} \mathfrak{F}_i)\} \quad (13)$$

$$\mathcal{H} = \bigodot_{\forall\, i\, \in \mathcal{A}}\{(\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i) \wedge_{\mathcal{A}} (\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i \rightarrow_{\mathcal{A}} \mathfrak{F}_i)\} \quad (14)$$

A brief explanation of the above equations can be elaborated as both of these equations are formulated for the constraints (key factors) in the set $\mathcal{A}$, which play a crucial role in impacting the financial threat evaluation. The (13) enables us to prevent the actual values from being aggregated too lowly while giving us a mathematical technique to collectively assess the constraints and provide a single robust value. Additionally, the (14) calculates the effectiveness of all the constraints on financial risk assessment while minimizing the error. Also, I-VFNNs will be employed to reduce the ambiguity and imprecision from the evaluation of the impact of artificial intelligence in financial risk assessment.

In addition, let us explain that $\wedge_{\mathcal{A}}$ is the I-VFN conjunction, $\rightharpoondown_{\mathcal{A}}$ is the I-VFN implication, $\sqcup_{\forall\, i\in\mathcal{A}}\mathfrak{T}=\wedge_{\mathcal{A}_i}\mathfrak{T}$ and $\odot_{\forall\, i\in\mathcal{A}}\mathfrak{T}=\vee_{\mathcal{A}_i}\mathfrak{T}$, where $\mathfrak{T}$ is any I-VFNN.

Let us note that the I-VFN norm

$$\left\{\begin{matrix}(\mathcal{T}_1^M,\mathcal{T}_1^N),(\mathcal{J}_1^M,\mathcal{J}_1^N),\\ (\mathcal{F}_1^M,\mathcal{F}_1^N)\end{matrix}\right\}\wedge_{\mathcal{A}}\left\{\begin{matrix}(\mathcal{T}_2^M,\mathcal{T}_2^N),(\mathcal{J}_2^M,\mathcal{J}_2^N),\\ (\mathcal{F}_2^M,\mathcal{F}_2^N)\end{matrix}\right\} = \left\{\begin{matrix}\left(min(\mathcal{T}_1^M,\mathcal{T}_2^M),min(\mathcal{T}_1^N,\mathcal{T}_2^N)\right),\\ \left(max(\mathcal{J}_1^M,\mathcal{J}_2^M),max(\mathcal{J}_1^N,\mathcal{J}_2^N)\right),\\ \left(max(\mathcal{F}_1^M,\mathcal{F}_2^M),max(\mathcal{F}_1^N,\mathcal{F}_2^N)\right)\end{matrix}\right\}$$

the I-VFN co-norm

$$\left\{\begin{matrix}(\mathcal{T}_1^M,\mathcal{T}_1^N),(\mathcal{J}_1^M,\mathcal{J}_1^N),\\ (\mathcal{F}_1^M,\mathcal{F}_1^N)\end{matrix}\right\}\vee_{\mathcal{A}}\left\{\begin{matrix}(\mathcal{T}_2^M,\mathcal{T}_2^N),(\mathcal{J}_2^M,\mathcal{J}_2^N),\\ (\mathcal{F}_2^M,\mathcal{F}_2^N)\end{matrix}\right\} = \left\{\begin{matrix}\left(max(\mathcal{T}_1^M,\mathcal{T}_2^M),max(\mathcal{T}_1^N,\mathcal{T}_2^N)\right),\\ \left(min(\mathcal{J}_1^M,\mathcal{J}_2^M),min(\mathcal{J}_1^N,\mathcal{J}_2^N)\right),\\ \left(min(\mathcal{F}_1^M,\mathcal{F}_2^M),min(\mathcal{F}_1^N,\mathcal{F}_2^N)\right)\end{matrix}\right\}$$

the I-VFN negation

$$\rightharpoondown_N\left\{\begin{matrix}(\mathcal{T}^M,\mathcal{T}^N),(\mathcal{J}^M,\mathcal{J}^N),\\ (\mathcal{F}^M,\mathcal{F}^N)\end{matrix}\right\}=\left\{\begin{matrix}(\mathcal{F}^M,\mathcal{F}^N),(\mathcal{J}^M,\mathcal{J}^N),\\ (\mathcal{T}^M,\mathcal{T}^N)\end{matrix}\right\},$$

and I-VFN implication

$$\left\{\begin{matrix}(\mathcal{T}_1^M,\mathcal{T}_1^N),(\mathcal{J}_1^M,\mathcal{J}_1^N),\\ (\mathcal{F}_1^M,\mathcal{F}_1^N)\end{matrix}\right\}\rightharpoondown_{\mathcal{A}}\left\{\begin{matrix}(\mathcal{T}_2^M,\mathcal{T}_2^N),(\mathcal{J}_2^M,\mathcal{J}_2^N),\\ (\mathcal{F}_2^M,\mathcal{F}_2^N)\end{matrix}\right\} =\rightharpoondown_N\left\{\begin{matrix}(\mathcal{T}_1^M,\mathcal{T}_1^N),(\mathcal{J}_1^M,\mathcal{J}_1^N),\\ (\mathcal{F}_1^M,\mathcal{F}_1^N)\end{matrix}\right\}\vee_{\mathcal{A}}\left\{\begin{matrix}(\mathcal{T}_2^M,\mathcal{T}_2^N),(\mathcal{J}_2^M,\mathcal{J}_2^N),\\ (\mathcal{F}_2^M,\mathcal{F}_2^N)\end{matrix}\right\}$$

are used.

IV. NUMERICAL STUDY

In the previous section, we have proposed a novel mathematical model using I-VFNNs to evaluate the impact of artificial intelligence on financial risk assessment; therefore, to prove its numerical and practical applicability, we are going to solve a numerical example in this section of the research article.

Example 1: Consider that we are evaluating the impact of AI on financial risk assessment while assuming the constraints in Table 1 namely $\mathfrak{R}_i$, $\mathfrak{F}_i$ and $\mathfrak{C}_i$, also assume that we have taken three different cases from the set $\mathcal{A}$ to assess the implacability and four experts, assume $\mathfrak{X}=\{\mathfrak{X}_1,\mathfrak{X}_2,\mathfrak{X}_3,\mathfrak{X}_4\}$, will given their judgements in the form of numerical data. The cases are chosen randomly as $\mathcal{A}=\{\mathcal{A}_{38},\mathcal{A}_{41},\mathcal{A}_{50}\}$, so we can show them in the form of an ordered triple of the variables as

$$\mathcal{A}_{38}=(\mathfrak{R}_{38},\mathfrak{F}_{38},\mathfrak{C}_{38})$$

$$\mathcal{A}_{41}=(\mathfrak{R}_{41},\mathfrak{F}_{41},\mathfrak{C}_{41})$$

$$\mathcal{A}_{50}=(\mathfrak{R}_{50},\mathfrak{F}_{50},\mathfrak{C}_{50})$$

The proposed method can be applied to multiple cases at a time, which shows its flexibility. We have taken each of the expert financial threat managers with a high or very high level of expertise to get a more accurate, robust, and precise solution to the issue. The weights according to each of the experts are given in Table 3.

Also, the numerical information given by all experts on the cases under study is shown in the Table 4 where constraints are represented in the form of interval-valued Fermatean Neutrosophic Numbers (I-VFNNs) along with the aggregated values using (10), (11), and (12).

Now we are going to find the mathematical components in (13) and (14) for each of the constraints, as given in Table 5. Therefore, we have our final results from (13) and (14) as,

$$\mathcal{E}=\{(0.22,0.41),(0.65,0.64),(0.75,0.76)\}$$

and

$$\mathcal{H}=\{(0.34,0.44),(0.53,0.40),(0.59,0.45)\}$$

We can defuzzify these I-VFN values to real numerical values using Definition 10. yielding us the results $\mathcal{E}=0.7386$ and $\mathcal{H}=0.3169$.

Table 3. Weights Allotted to the Experts

Expert	Expertise level	Weights $(\mathfrak{w}_j)$
$\mathfrak{X}_1$	High	0.73
$\mathfrak{X}_2$	Very high	0.9
$\mathfrak{X}_3$	High	0.73
$\mathfrak{X}_4$	Very high	0.9

Table 4. Initial Data Gathered from the Experts Along with the Evaluated Values

Expert / Variable	$\mathfrak{R}_{38}$	$\mathfrak{F}_{38}$	$\mathfrak{C}_{38}$	$\mathfrak{R}_{41}$	$\mathfrak{F}_{41}$	$\mathfrak{C}_{41}$	$\mathfrak{R}_{50}$	$\mathfrak{F}_{50}$	$\mathfrak{C}_{50}$
$\mathfrak{X}_1$	{(0.55,0.74), (0.27,0.48), (0.82, 0.16)}	{(0.44,0.64), (0.12,0.66), (0.94,0.56)}	{(0.74,0.32), (0.74,0.64), (0.63,0.32)}	{(0.16,0.79), (0.71,0.17), (0.53,0.78)}	{(0.61,0.27), (0.53,0.45), (0.72,0.87)}	{(0.16,0.83), (0.38,0.50), (0.13,0.55)}	{(0.43,0.15), (0.91,0.35), (0.78,0.16)}	{(0.72,0.28), (0.31,0.32), (0.65,0.67)}	{(0.88,0.11), (0.41,0.02), (0.44,0.08)}
$\mathfrak{X}_2$	{(0.61,0.45), (0.47,0.86), (0.49,0.53)}	{(0.54,0.95), (0.74,0.35), (0.17,0.61)}	{(0.72,0.18), (0.34,0.76), (0.39,0.37)}	{(0.58,0.22), (0.67,0.33), (0.77,0.85)}	{(0.83,0.26), (0.85,0.86), (0.59,0.62)}	{(0.17,0.78), (0.38,0.50), (0.71,0.48)}	{(0.16,0.66), (0.77,0.64), (0.96,0.66)}	{(0.29,0.56), (0.35,0.96), (0.23,0.65)}	{(0.27,0.86), (0.36,0.68), (0.82,0.57)}
$\mathfrak{X}_3$	{(0.01,0.47), (0.55,0.33), (0.46,0.44)}	{(0.41,0.02), (0.55,0.22), (0.02,0.41)}	{(0.50,0.38), (0.77,0.56), (0.41,0.45)}	{(0.11,0.12), (0.35,0.11), (0.69,0.96)}	{(0.75,0.56), (0.47,0.88), (0.71,0.61)}	{(0.21,0.12), (0.78,0.44), (0.83,0.81)}	{(0.10,0.70), (0.56,0.54), (0.38,0.04)}	{(0.86,0.49), (0.54,0.76), (0.68,0.94)}	{(0.58,0.91), (0.36,0.84), (0.58,0.58)}
$\mathfrak{X}_4$	{(0.16,0.81), (0.37,0.32), (0.61,0.63)}	{(0.74,0.47), (0.19,0.23), (0.47,0.47)}	{(0.37,0.73), (0.48,0.48), (0.63,0.49)}	{(0.83,0.63), (0.40,0.28), (0.38,0.50)}	{(0.88,0.33), (0.94,0.09), (0.50,0.02)}	{(0.70,0.49), (0.16,0.19), (0.59,0.64)}	{(0.19,0.84), (0.39,0.73), (0.81,0.32)}	{(0.30,0.37), (0.58,0.94), (0.03,0.46)}	{(0.59,0.06), (0.18,0.95), (0.09,0.25)}
Aggregated Values	{(0.34,0.62), (0.42,0.51), (0.59,0.45)}	{(0.54,0.54), (0.41,0.36), (0.39,0.51)}	{(0.58,0.41), (0.56,0.61), (0.51,0.41)}	{(0.45,0.44), (0.53,0.23), (0.59,0.76)}	{(0.77,0.35), (0.72,0.56), (0.62,0.51)}	{(0.32,0.56), (0.41,0.40), (0.57,0.61)}	{(0.22,0.60), (0.65,0.58), (0.75,0.32)}	{(0.52,0.43), (0.45,0.77), (0.37,0.67)}	{(0.56,0.48), (0.32,0.64), (0.48,0.37)}

Example 2: Consider that we are taking the data from *Example 1:* Except the weights of experts. New weights of experts for this example are given in Table 6.

Using (10), (11), and (12) on the information gathered by the experts given in Table 4. We get the aggregated values in Table 7.

Now we are going to find the mathematical components in (13) and (14) for each of the constraints, as given in Table 8.

Table 5. Evaluation of Components of (13) and (14) for Example 1

Variables	$\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i$	$\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i \rightarrow_{\mathcal{A}} \mathfrak{F}_i$	$(\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i) \wedge_{\mathcal{A}} (\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i \rightarrow_{\mathcal{A}} \mathfrak{F}_i)$
$\mathcal{A}_{38}$	$\{(0.34,0.41),(0.56,0.61),(0.59,0.45)\}$	$\{(0.59,0.54),(0.41,0.36),(0.34,0.41)\}$	$\{(0.34,0.41),(0.56,0.61),(0.59,0.45)\}$
$\mathcal{A}_{41}$	$\{(0.32,0.44),(0.53,0.40),(0.59,0.76)\}$	$\{(0.77,0.76),(0.53,0.40),(0.32,0.44)\}$	$\{(0.32,0.44),(0.53,0.40),(0.59,0.76)\}$
$\mathcal{A}_{50}$	$\{(0.22,0.48),(0.65,0.64),(0.75,0.37)\}$	$\{(0.75,0.43),(0.45,0.64),(0.22,0.48)\}$	$\{(0.22,0.43),(0.65,0.64),(0.75,0.48)\}$

Table 6. Weights Allotted to the Experts

Expert	Expertise level	Weights $(\mathfrak{w}_j)$
$\mathfrak{x}_1$	High	0.73
$\mathfrak{x}_2$	Very low	0.2
$\mathfrak{x}_3$	Moderate	0.55
$\mathfrak{x}_4$	Low	0.36

Table 7. Aggregated Values for Example 2

$\mathfrak{R}_{38}$	$\mathfrak{F}_{38}$	$\mathfrak{C}_{38}$
$\{(0.32,0.64),(0.40,0.44),(0.64,0.38)\}$	$\{(0.50,0.46),(0.33,0.41),(0.49,0.50)\}$	$\{(0.59,0.40),(0.65,0.60),(0.54,0.40)\}$
$\mathfrak{R}_{41}$	$\mathfrak{F}_{41}$	$\mathfrak{C}_{41}$
$\{(0.32,0.50),(0.54,0.19),(0.57,0.79)\}$	$\{(0.73,0.37),(0.63,0.55),(0.66,0.60)\}$	$\{(0.28,0.54),(0.46,0.42),(0.49,0.64)\}$
$\mathfrak{R}_{50}$	$\mathfrak{F}_{50}$	$\mathfrak{C}_{50}$
$\{(0.26,0.50),(0.69,0.51),(0.68,0.21)\}$	$\{(0.63,0.39),(0.44,0.64),(0.49,0.71)\}$	$\{(0.67,0.42),(0.34,0.52),(0.45,0.32)\}$

Table 8. Evaluation of Components of (13) and (14) for Example 2

Variables	$\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i$	$\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i \rightarrow_{\mathcal{A}} \mathfrak{F}_i$	$(\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i) \wedge_{\mathcal{A}} (\mathfrak{R}_i \wedge_{\mathcal{A}} \mathfrak{C}_i \rightarrow_{\mathcal{A}} \mathfrak{F}_i)$
$\mathcal{A}_{38}$	$\{(0.32,0.40),(0.65,0.60),(0.64,0.40)\}$	$\{(0.64,0.46),(0.33,0.41),(0.32,0.40)\}$	$\{(0.32,0.40),(0.65,0.60),(0.64,0.40)\}$
$\mathcal{A}_{41}$	$\{(0.28,0.50),(0.54,0.42),(0.57,0.79)\}$	$\{(0.73,0.79),(0.54,0.42),(0.28,0.50)\}$	$\{(0.28,0.50),(0.54,0.42),(0.57,0.79)\}$
$\mathcal{A}_{50}$	$\{(0.26,0.42),(0.69,0.52),(0.68,0.32)\}$	$\{(0.68,0.39),(0.44,0.52),(0.26,0.42)\}$	$\{(0.26,0.39),(0.69,0.52),(0.68,0.42)\}$

Therefore, we have our final results from (13) and (14) as,

$$\mathcal{E} = \{(0.26,0.39),(0.69,0.60),(0.68,0.79)\}$$

and

$$\mathcal{H} = \{(0.32,0.50),(0.54,0.42),(0.57,0.40)\}$$

We can defuzzify these I-VFN values to real numerical values using *Definition 10.* yielding us the results $\mathcal{E} = 0.7144$ and $\mathcal{H} = 0.3192$.

A. Discussion

The results of both examples are not too high or too low, and show that, according to the given information, the impact of AI on financial risk evaluation is more or less moderate. Although the study shows that artificial intelligence has significant impacts on financial risk assessment while preventing threats like fraudulent

all know, artificial intelligence is modernizing the world, specifically in financial risk assessment, but in the end, human intelligence and decision-making skills exceed its revolution. All in all, AI should be used in different areas but on a moderate level, to make its good use as much as we can, and prevent any misconceptions and misconduct through full control of the human mind.

V. Comparative Study

Here, we are going to make a comparison of the proposed approach to the existing models to present the superiority of the suggested technique. A similar technique has been used previously using neutrosophic sets (NSs) for assessing the impact of AI on civil liability [39]. But since the neutrosophic sets only consider the membership factors of truth, indeterminacy, and falsity grades, they neglect the factors of non-membership of all of these grades. Hence, they lack the ability to properly assess the intricate real-world situations. The numerical comparison of our results with the results presented in [33] is given in Table 9.

The results aggregated through neutrosophic sets are not clear and do not show any proper difference between them, presenting the average values. But the proposed technique shows the proper variations and accurate assessment of the at-hand situation. The pictorial comparison of these techniques is given in Fig. 5 and Fig. 6.

Table 9. Comparison of the Aggregated Results

Parameters	$\mathcal{E}$	$\mathcal{H}$
NSs	0.503	0.583
I-VFNSs Example 1	0.7386	0.3169
I-VFNSs Example 2	0.7144	0.3192

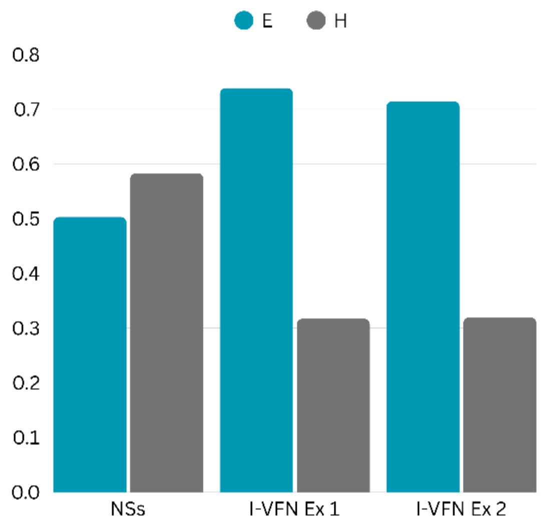

Fig. 5. Bar chart of the comparisons

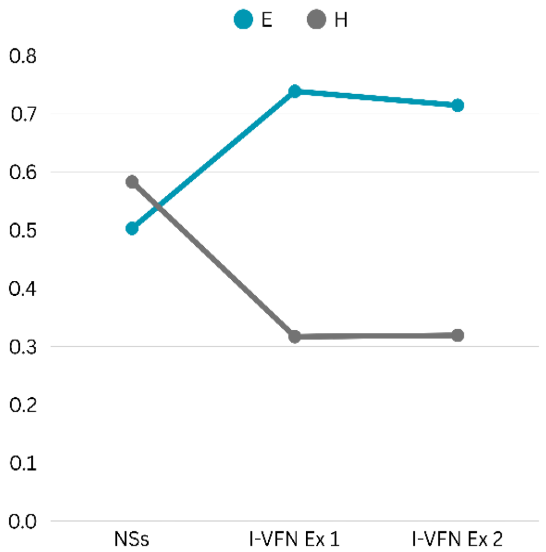

Fig. 6. Line chart of the comparisons

VI. CONCLUSION

The stability and advancement of economies depend upon financial risk assessment, which is a complicated, and variable system. Conventional financial models no longer have potentiality in the present fast-paced modern economy due to simple presumptions and restricted potential. Therefore, a new chapter is unlocked by Artificial Intelligence (AI), which highlights a significant change in the policies of recognizing, forecasting, and managing the financial risks. In this research article, we also introduced a scientific and cryptic analysis of the challenges presented by Artificial Intelligence (AI), apart from the function and influence of AI in financial risk assessment. AI is making financial risk management more precise through automated monitoring, flexible risk modeling, and actioning real-time data. AI is useful for not just credit rating but also evaluating risks related to customer behaviors, simulating scenarios, and recognizing fraudulent activity. However, these models periodically have to give up some ambiguity and uncertainty that comes with being in the modern world. These challenges can be addressed through interval-valued fuzzy neutrosophic sets (I-VFNSs), because they view the truth, indeterminacy, and falsity of the information as a spectrum, which can clarify financial risk assessments. This process will also enable us to identify very (extremely) subtle alterations in finance, so more accurate forecasting and proper planning models will be commonplace. I-VFN neural networks will classify, assess worst-case scenarios, and, in realistic realm, evaluate the composite effect of AI factors. Both I-VFNNs and AI provide the extent of that threat, and its likelihood of occurrence, and provide an early indication of the location. Financial decisions relying on uncertain information from I-VFNNS are now enhanced and prepared for the future. By detecting fraud quickly and understanding the complex trends through I-VFNNs and AI, we can lower the amount of money lost. These automated systems, in addition to enabling us to source, predict, and replicate threats, integrate the old problem of obfuscation or uncertainty in a much greater way. If the financial institutions embrace the recommendations in this paper, they will gain value and protect their assets, and long-last as a valuable institution in the financial markets. In the future, the incorporation of advanced mathematics tools, like I-VFNNs and AI, will provide us with ready, innovative, and more stable financial strategies to improve the sustainability and offerings for both companies and society.

Even though this study represents a significant step forward, it is not without limitations, which are, the use of I-VFNNs involves complex sensing processes that require significant computational resources, there is an ongoing requirement for the training, and updating of an AI model, requiring continuous work, time and privacy of data, and Biases or errors that exist in financial data may impact the results of an AI model, and thus, the performance of an I-VFNN may also be affected.

In the future, more research may be done on how I-VFNNs, can be integrated with hybrid neural networks, can be incorporated into super fuzzy logic or nano-computing, and can be part of automated response systems in real-time.

REFERENCES

[1] M. Fahrezi, "A Systematic Literature Review: The Use of Artificial Intelligence and Machine Learning in Financial Risk Management and Predictive Analytics," *International Journal of Research and Applied Technology*, vol. 4, no. 1, pp. 211–223, 2024, https://ojs.unikom.ac.id/index.php/injuratech/article/view/16822.

[2] C. Oko-Odion, "AI-Driven Risk Assessment Models for Financial Markets: Enhancing Predictive Accuracy and Fraud Detection," *International Journal of Computer Applications Technology and Research*, vol. 14, pp. 80–96, 2025, https://doi.org/10.7753/ijcatr1404.1007.

[3] D. Aikman *et al.*, "Taking uncertainty seriously: Simplicity versus complexity in financial regulation," *Industrial and Corporate Change*, vol. 30, no. 2, pp. 317–345, 2021, https://doi.org/10.1093/icc/dtaa024.

[4] Y. Sari and A. Indrabudiman, "The Role of Artificial Intelligence (AI) in Financial Risk Management," *Formosa Journal of Sustainable Research*, vol. 3, no. 9, pp. 2073–2082, 2024, https://doi.org/10.55927/fjsr.v3i9.11436.

[5] L. D. Oyeniyi, C. E. Ugochukwu, and N. Z. Mhlongo, "Transforming financial planning with AI-driven analysis: a review and application insights," *Finance & Accounting Research Journal*, vol. 6, no. 4, pp. 626–647, 2024, https://doi.org/10.51594/farj.v6i4.1037.

[6] S. Broumi, R. Sundareswaran, M. Shanmugapriya, P. K. Singh, M. Voskoglou, and M. Talea, "Faculty Performance Evaluation through Multi-Criteria Decision Analysis Using Interval-Valued Fermatean Neutrosophic Sets," *Mathematics*, vol. 11, no. 18, pp. 3817–3821, 2023, https://doi.org/10.3390/math11183817.

[7] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965, https://doi.org/10.1016/S0019-9958(65)90241-X.

[8] P. Chotikunnan, W. Khotakham, P. Imura, R. Chotikunnan, A. Wongkamhang, and N. Thongpance, "Comparative Analysis of PID-Driven Data-Based and PSO-Tuned Fuzzy Membership Functions for Robotic Manipulator Control," *Journal of Fuzzy Systems and Control*, vol. 3, no. 3, pp. 204–211, 2025, https://doi.org/10.59247/jfsc.v3i3.335.

[9] K. Bhalla, D. Koundal, B. Sharma, Y. C. Hu, and A. Zaguia, "A fuzzy convolutional neural network for enhancing multi-focus image fusion," *Journal of Visual Communication and Image Representation*, vol. 84, p. 103485, 2022, https://doi.org/10.1016/j.jvcir.2022.103485.

[10] N. X. Chiem and N. C. B. Nguyen, "Design of Embedded Control System with Fuzzy Controller and Nonlinear Controller for the Line Follower Robot," *Journal of Fuzzy Systems and Control*, vol. 3, no. 2, pp. 122–127, 2025, https://doi.org/10.59247/jfsc.v3i2.303.

[11] M. Kamran *et al.*, "A Systematic Formulation into Neutrosophic Z Methodologies for Symmetrical and Asymmetrical Transportation Problem Challenges," *Symmetry*, vol. 16, no. 5, p. 615, 2024, https://doi.org/10.3390/sym16050615.

[12] F. A. Azeez and B. J. Hamza, "Optimizing Hybrid LiFi Communication Systems Using Fuzzy Reinforcement Learning for Enhanced Network Performance," *Journal of Fuzzy Systems and Control*, vol. 3, no. 2, pp. 170–173, 2025, https://doi.org/10.59247/jfsc.v3i2.316.

[13] D. K. Sudhish, L. R. Nair, and S. S, "Content-based image retrieval for medical diagnosis using fuzzy clustering and deep learning," *Biomedical Signal Processing and Control*, vol. 88, p. 105620, 2024, https://doi.org/10.1016/j.bspc.2023.105620.

[14] H.-T. Nguyen *et al.*, "Experiment Ball Levitation with Fuzzy PID and PID Implementation," *Journal of Fuzzy Systems and Control*, vol. 2, no. 3, pp. 129–134, 2024, https://doi.org/10.59247/jfsc.v2i3.221.

[15] J. Więckowski, B. Kizielewicz, A. Shekhovtsov, and W. Sałabun, "How Do the Criteria Affect Sustainable Supplier Evaluation?-A Case Study Using Multi-Criteria Decision Analysis Methods in a Fuzzy Environment," *Journal of Engineering Management and Systems Engineering*, vol. 2, no. 1, pp. 37–52, 2023, https://doi.org/10.56578/jemse020102.

[16] A. Uzair, A. Ishtiaq, and M. Rasheed, "Non-kekulean benzenoid hydrocarbon: physico-chemical properties for benzenoid hydrocarbon using topological indices and m-polynomial," *Computational Algorithms and Numerical Dimensions*, vol. 3, pp. 94–114, 2024, https://doi.org/10.22105/cand.2024.470538.1100.

[17] S. H. Omran, S. W. Shneen, M. H. Ali, Q. A. Jawad, S. A. Gitaffa, and H. M. Salman, "Comparative Traditional Methods of Attributes with Fuzzy Quality Control Charts for Improving the Quality of a Product," *Journal of Fuzzy Systems and Control*, vol. 3, no. 1, pp. 22–29, 2024, https://doi.org/10.59247/jfsc.v3i1.273.

[18] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986, https://doi.org/10.1016/S0165-0114(86)80034-3.

[19] B. Gohain, R. Chutia, and P. Dutta, "Distance measure on intuitionistic fuzzy sets and its application in decision-making, pattern recognition, and clustering problems," *International Journal of Intelligent Systems*, vol. 37, no. 3, pp. 2458–2501, 2022, https://doi.org/10.1002/int.22780.

[20] A. Ishtiaq, K. Tul Kubra, A. Uzair, and A. Ali, "Quantifying Multi-Cause Psychological Disorder Risk Through an Advanced Mathematical Model Using Intuitionistic Pentagonal Fuzzy Logic," *Spectrum of Operational Research*, pp. 1–21, 2025, https://doi.org/10.31181/sor41202762.

[21] X. Peng and Y. Yang, "Some Results for Pythagorean Fuzzy Sets," *International Journal of Intelligent Systems*, vol. 30, no. 11, pp. 1133–1160, 2015, https://doi.org/10.1002/int.21738.

[22] A. Kumar Adak, D. Kumar, and S. A. Edalatpanah, "Some New Operations on Pythagorean Fuzzy Sets," *Uncertainty Discourse and Applications*, vol. 1, no. 1 SE-Articles, pp. 11–19, 2024, https://doi.org/10.48313/uda.v1i1.17.

[23] M. Akram, G. Muhammad, and D. Ahmad, "Analytical solution of the Atangana–Baleanu–Caputo fractional differential equations using Pythagorean fuzzy sets," *Granular Computing*, vol. 8, no. 4, pp. 667–687, 2023, https://doi.org/10.1007/s41066-023-00364-3.

[24] T. Senapati and R. R. Yager, "Fermatean fuzzy sets," *Journal of Ambient Intelligence and Humanized Computing*, vol. 11, no. 2, pp. 663–674, 2020, https://doi.org/10.1007/s12652-019-01377-0.

[25] J. Chakraborty, S. Mukherjee, and L. Sahoo, "An Alternative Approach for Enhanced Decision-Making using Fermatean Fuzzy Sets," *Spectrum of Engineering and Management Sciences*, vol. 2, no. 1, pp. 135–150, 2024, https://doi.org/10.31181/sems21202411j.

[26] M. Kirişci, "New cosine similarity and distance measures for Fermatean fuzzy sets and TOPSIS approach," *Knowledge and Information Systems*, vol. 65, no. 2, pp. 855–868, 2023, https://doi.org/10.1007/s10115-022-01776-4.

[27] F. Smarandache, *A Unifying Field in Logics: Neutrosophic Logic, Neutrosophic Set, Neutrosophic Probability and Statistics (fourth edition)*. Rehoboth, NM, USA: American Research Press, 2001. http://arxiv.org/abs/math/0101228.

[28] M. Akram, G. Ali, J. C. R. Alcantud, and A. Riaz, "Group decision-making with Fermatean fuzzy soft expert knowledge," *Artificial Intelligence Review*, vol. 55, no. 7, pp. 5349–5389, 2022, https://doi.org/10.1007/s10462-021-10119-8.

[29] C. Delcea, A. Domenteanu, C. Ioanăș, V. M. Vargas, and A. N. Ciucu-Durnoi, "Quantifying Neutrosophic Research: A Bibliometric Study," *Axioms*, vol. 12, no. 12, p. 1083, 2023, https://doi.org/10.3390/axioms12121083.

[30] S. Broumi, R. Sundareswaran, M. Shanmugapriya, A. Bakali, and M. Talea, "Theory and Applications of Fermatean Neutrosophic Graphs," *Neutrosophic Sets and Systems*, vol. 50, p. 2022, 2022, https://digitalrepository.unm.edu/nss_journal/vol50/iss1/15/.

[31] M. Saeed, M. U. Nisa, M. H. Saeed, T. Alballa, and H. A. E. W. Khalifa, "Detecting Patterns of Infection-Induced Fertility Using Fermatean Neutrosophic Set With Similarity Analysis," *IEEE Access*, vol. 11, pp. 112320–112333, 2023, https://doi.org/10.1109/ACCESS.2023.3323024.

[32] N. Gonul Bilgin, D. Pamučar, and M. Riaz, "Fermatean Neutrosophic Topological Spaces and an Application of Neutrosophic Kano Method," *Symmetry*, vol. 14, no. 11, 2022, https://doi.org/10.3390/sym14112442.

[33] S. Dhouib, K. Vidhya, S. Broumi, and M. Talea, "Solving the Minimum Spanning Tree Problem Under Interval-Valued Fermatean Neutrosophic Domain," *Neutrosophic Sets and Systems*, vol. 67, pp. 11–20, 2024, https://doi.org/10.5281/zenodo.11098903.

[34] M. Kamran, M. Nadeem, J. Żywiołek, M. E. M. Abdalla, A. Uzair, and A. Ishtiaq, "Enhancing Transportation Efficiency with Interval-Valued Fermatean Neutrosophic Numbers: A Multi-Item Optimization Approach," *Symmetry*, vol. 16, no. 6, 2024, https://doi.org/10.3390/sym16060766.

[35] Y. Cai, "Interval Valued Fermatean Neutrosophic for Analysis Risk Management of Practical Teaching Quality in University Art and Design Programs," *Neutrosophic Sets and Systems*, vol. 83, pp. 27–49, 2025, https://doi.org/10.5281/zenodo.15122386.

[36] M. E. M. Abdalla, A. Uzair, A. Ishtiaq, M. Tahir, and M. Kamran, "Algebraic Structures and Practical Implications of Interval-Valued Fermatean Neutrosophic Super HyperSoft Sets in Healthcare," *Spectrum of Operational Research*, vol. 2, no. 1, pp. 240–259, 2025, https://doi.org/10.31181/sor21202523.

[37] M. Saranya and D. Jayanthi, "Novel Cosine Similarity Measures for Interval-Valued Fermatean Neutrosophic Sets with TOPSIS-Based MCDM Applications," *Neutrosophic Sets and Systems*, vol. 93, pp. 44–65, 2025, https://doi.org/https://doi.org/10.5281/zenodo.16903431.

[38] B. Ashtiani, F. Haghighirad, A. Makui, and G. A. Montazer, "Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets," *Applied Soft Computing Journal*, vol. 9, no. 2, pp. 457–461, Mar. 2009, https://doi.org/10.1016/j.asoc.2008.05.005.

[39] M. G. Paredes-Morales, G. J. Alberdy-Rodriguez, S. G. Morales-Cobos, D. A. Castro-Valderrama, G. A. Valderrama-Barragán, and E. F. Valderrama-Barragán, "A Neutrosophic Model for Assessing the Impact of Artificial Intelligence on Civil Liability," *Neutrosophic Sets and Systems*, vol. 81, pp. 382–388, 2025, https://digitalrepository.unm.edu/nss_journal/vol81/iss1/23/.